

716/Math. UG/6th Sem/MATH-H-DSE-T-04B/23

U.G. 6th Semester Examination - 2023

MATHEMATICS

[HONOURS]

**Discipline Specific Elective (DSE)
Course Code : MATH-H-DSE-T-04B**

(Biomathematics)

Full Marks : 60 Time : 2½ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

The symbols and notations have their usual meanings.

1. Answer any ten questions: $2 \times 10 = 20$
- a) State the basic equations of classical Lotka-Volterra model for a prey-predator system.
 - b) Define the term 'subcritical Hopf'.
 - c) What is bifurcation?
 - d) What is non-hyperbolic equilibrium point of a dynamical system?
 - e) State Hartman-Grobman theorem.
 - f) State Routh-Hurwitz criterion.

[Turn Over]

- g) Define non-autonomous system of a dynamical system. 5
- h) Define phase-space in autonomous system.
- i) What is activator-inhibitor system?
- j) Define the term 'Diffusive instability'.
- k) State Bendixson's negative criterion.
- l) What is allee effect in biological system?
- m) Define the terminology 'Holling type growth'.
- n) Define eco-epidemiology.

2. Answer any four questions: 5 × 4 = 20

- a) State with proper justifications how a cardiovascular system can be analysed as a mathematical model. 5
- b) Consider the system $\frac{dX}{dt} = Y + \alpha X$, $\frac{dY}{dt} = -2X$, check the stability of the system for the different values of α . 5
- c) Find the Lyapunov function for the system of equations

$$\frac{dX}{dt} = -5Y - 2X^3, \quad \frac{dY}{dt} = 5X - 3Y^3.$$

- d) State the assumptions and formulate the simple epidemic (SIR) model. 5
- e) Describe a host-parasitoid system with all state variables and parameters. Find the equilibrium point of the model. 3+2
- f) Formulate a ratio-dependent eco-epidemiological model with proper assumptions. 5

3. Answer any two questions: 10 × 2 = 20

- a) Consider the system: $\frac{dX}{dt} = rX \left(1 - \frac{X}{K} \right) - H$. Find the equilibrium points and check the stability of the system around the equilibrium points. Also find the critical value of H for a bifurcation to occur. 3+4+3

- b) Formulate a predator-prey model with disease on prey population. State the necessary assumptions of the proposed model. Find the conditions for which infection-free and predator-free equilibrium are locally asymptotically stable. 3+2+5

- c) Define asymptotic stability of an equilibrium point. State Dulac's criterion. Show that the system

$$\frac{dX}{dt} = aX - bXY, \quad \frac{dY}{dt} = cXY - dY, \quad \text{where}$$

$$a, b, c, d > 0, \quad X(0) = 0, \quad Y(0) = 0 \quad \text{is}$$

conservative and each solution $(X(t), Y(t))$ is a periodic solution. 2+2+6

- d) Write short notes on: 5+5

- i) Basic reproduction number
- ii) Logistic growth and Gompertz growth

Full Marks : 40

The figures in the right-hand margin

Candidates are required to give their own words as far as possible.

I. Answer any five of the following

- a) What is meant by plagium?
- b) What does molal solution mean?
- c) What is meant by serial dilution?
- d) A sample having diameter 100 micrometers, you to observe under microscope you should follow, What should you do? Justify.
- e) Write the usefulness of microtome in writing.
- f) What is 'microtome'? Write its parts.